Bachelor of Arts (B.A.) Part—I (Semester—I) Examination

STATISTICS

Optional Paper—1

(Probability Theory)

Time : Three Hours] [Maximum Marks : 50

N.B. :— ALL questions are compulsory and carry equal marks.

1. (A) Define the following giving one example of each :
   (i) Complementary event
   (ii) Elementary event
   (iii) Impossible event
   (iv) Mutually exclusive events
   (v) Exhaustive events.

   State the axiomatic definition of probability. Using this definition prove the following results :
   (i) Probability of an impossible event is zero.
   (ii) P(S) = 1, where S is the sample space.
   (iii) $P(\bar{A}) = 1 - P(A)$. 10

OR

(E) There are 8 bulbs in the stock of a shop, of which 3 are defective. The shopkeeper on customer's demand, picks up 2 bulbs randomly. What is the probability that both the bulbs are defective ?

(F) Let A, B and C be three events in the sample space. Find expressions as union and/or intersection of these events in the following cases :
   (i) At least one of three events occur
   (ii) A occurs with either B or C
   (iii) A and B occur but C does not occur.

(G) Give classical definition of probability. State its limitations.

(H) A, B and C are three mutually exclusive and exhaustive events.

   Find P(B), if $\frac{1}{3}$ P(C) = $\frac{1}{2}$P(A) = P(B). Also find $P(\bar{A} \cap \bar{B} \cap \bar{C})$. 2.5×4=10

2. (A) Define :

(i) Independent events

(ii) Conditional probability of event A given the event B.

Show that conditional probability satisfies, all the axioms of probability. State and prove the multiplicative law of probability for n events $A_1, A_2, \ldots, A_n$. 10

OR

(E) Define pair-wise and mutual independence of n events $A_1, A_2, \ldots, A_n$. An unbiased coin is tossed 3 times. A denotes the event that a head occurs on each of the first two tosses, B is the event that a tail occurs on the third toss and C is the event that exactly two tails occur in the 3 tosses. Check whether A, B and C are pair-wise independent or not.

(F) If the events $B_1, B_2, \ldots, B_n$ form a partition of the sample space with $P(B_i) \neq 0$ for $i = 1, 2, \ldots, n$, then for any event A in the sample space show that :

$$P(A) = \sum_{i=1}^{n} P(B_i) \cdot P(A \mid B_i).$$

The probability that it will be sunny tomorrow is 1/3. If it is sunny, the probability that Sania plays tennis is 4/5. The corresponding probability of playing tennis if it is not sunny is 2/5. What is the probability that Sania plays tennis ? 5+5

3. (A) Define the cumulative distribution function of a random variable. State and prove its properties.

If X is a r.v. with pdf
$$f(x) = \frac{1}{18}(6 - x) \quad , \quad 0 \le x \le 6$$
$$= 0 \quad , \quad \text{otherwise}$$

then find its cdf. Also find :

(i) $P[X > 2]$

(ii) $P[2 \le X \le 4]$. 10

OR

(E) Let a r.v. X has the pmf,

$$p(x) = P[X = x] = \frac{x}{15}, \; x = 1, 2, 3, 4, 5.$$

Find :

(i) cdf of X

(ii) $P[X > 3]$

(iii) $P[1 < X < 4]$.

(F) Let X be a r.v. with pdf

$$f(x) = 6x(1-x) \quad , \quad 0 < x < 1$$
$$= 0 \quad , \quad \text{otherwise}$$

Find :

(i) P[X < 1/4]

(ii) P[X > 1/2].

(G) Define expected value of a r.v.

Let X be a r.v. with cdf F(x) given by,

$$F(x) = \begin{cases} 0 & , \text{ for } x < -1 \\ \frac{x+1}{2} & , \text{ for } -1 \le x < 1 \\ 1 & , \text{ for } x \ge 1 \end{cases}$$

Find its pdf. Also find E(X).

(H) Let X be a r.v. with pdf f(x) given by

$$f(x) = \frac{1}{5} \quad , \quad \text{for } 2 < x < 7$$
$$= 0 \quad , \quad \text{elsewhere}$$

(i) Draw the graph of pdf.

(ii) Find P(3 < X < 5). 2.5×4=10

4. (A) Define probability generating function of a discrete r.v. Explain how the mean and the variance of the r.v. are obtained from its pgf. Obtain the pgf of $\frac{X-a}{b}$.

(B) Define median and mode of a r.v. Explain how these measures are calculated for a discrete and a continuous r.v.

Let X be a r.v. with pdf f(x) given by

$$f(x) = 3x^2 \quad , \quad 0 \le x \le 1$$
$$= 0 \quad , \quad \text{otherwise}$$

Find :

(i) Mean

(ii) Median

(iii) V(X). 5+5

OR

(E) Define the following for a r.v. :

(i) The r^{th} raw moment about A

(ii) The r^{th} raw moment about origin

(iii) The r^{th} central moment.

Derive the relationship for r^{th} central moment in terms of raw moments about origin. Hence obtain expressions for μ_2, μ_3 and μ_4. Let X be a r.v. with $\mu_1' = 2/3$, $\mu_2' = 1/2$ and $\mu_3' = 2/5$. Find μ_2 and μ_3. 10

5. Solve any ***ten*** out of the following questions :

(A) If A and B are exhaustive and mutually exclusive events then $P(A \cup B) =$ .... and $P(A \cap B) =$ ....

(B) If $P(A \cup B) = 4/5$ then find $P(\overline{A} \cap \overline{B})$.

(C) State the extension of addition law for n events A_1, A_2, ... A_n.

(D) Events A and B are such that,

$P(A) = 1/4$, $P(A \mid B) = 1/2$ and $P(B \mid A) = 2/3$.

Are A and B independent ?

(E) A fair die is thrown twice. What is the probability that the sum of two numbers at the upper faces is 6 given that no die shows a number '4' ?

(F) If A, B and C are 3 events then write the conditions for their mutual independence.

(G) Show that $E(cX) = cE(X)$ where c is a constant.

(H) A r.v. assumes values 1, 2 and 3 with $P[X \leq 2] = 2/3$. Find the pmf of X.

(I) Let X be a r.v. with pdf f(x), where

$$\begin{aligned} f(x) &= kx(2-x) \quad , \quad 0 \leq x \leq 2 \\ &= 0 \quad , \quad \text{otherwise} \end{aligned}$$

Find the value of k.

(J) Let X be a r.v. and c be a constant, then show that $V(cX) = c^2V(X)$. Hence state $V(4X + 5)$.

(K) If Karl Pearson's coefficient of skewness for a probability distribution is $\frac{1}{2}$ and the mean and mode are 5 and 2 respectively. Find the standard deviation.

(L) State the formula for the following measures :

(i) Measure of dispersion based on partition values.

(ii) Measure of skewness based on partition values. 1×10=10